Exercise: Algebraic Method

Lisa is a market researcher and plans to do a door-to-door survey to find out how many people use her company's products. She uses the following as a model for estimating the cost of the project:

$$C = 5000 + 80n$$

where n is the number of households questioned and C is the cost (in dollars).
Use this model to answer the following questions.

(a) If no households are surveyed, what is the cost of the project?

$C = 5000 + 80n = 5000 + 80 \times 0 = \$5{,}000$

(b) If 20 households are surveyed, what is the cost of the project?

$C = 5000 + 80n = 5000 + 80 \times 20 = \$\ 6{,}600$

(c) At one stage of the project Amina has questioned 40 households. How much more would it cost her to question another 10 households?

For $n = 40$
$C = 5000 + 80n = 5000 + 80 \times 40 = \$8{,}200$

For n = 50
$C = 5000 + 80n = 5000 + 80 \times 50 = \$9{,}000$

Extra cost is 9,000 – 8,200 = \$800

(d) Amina has been given a budget for the project of \$10000. Find the greatest number of households she can question.

$C = 5000 + 80n \Rightarrow 10000 = 5000 + 80n \Rightarrow 5000 = 80n \Rightarrow n = 62.5$
The greatest number of households she can question is 62.

(e) If Lisa could persuade her manager to increase the budget specified in part (d) by 15%, find the percentage increase in the greatest number of households she can question (round your answer to the nearest whole number).

10000 + 15% of 10000 = \$11,500
$C = 5000 + 80n \Rightarrow 11500 = 5000 + 80n \Rightarrow 6500 = 80n \Rightarrow n = 81.25$

The greatest number of households she can question is now 81.
Increase in the number of households is 81 – 62 = 19.
Percentage increase is $\frac{19}{81} \times 100\ \% = 23.45\ \% \approx 23\ \%$

Example 4

Adam expects the value of his car to depreciate according to the following linear model:

$$y = -11500t + 82000$$

where y is the value of Adam's car in dollars and t is the time in years.

(a) Use the model to find the time when the value of the car will be \$40000.

$y = -11500t + 82000 \Rightarrow 40000 = -11500t + 82000 \Rightarrow -42000 = -11500t$

$t = 3.65$ years

(b) Use the model to find the value of Adam's car:

(i) after 5 years

$y = -11500t + 82000 \Rightarrow y = -11500 \times 5 + 82000 \Rightarrow y =$ \$24500

(ii) after 15 years.

$y = -11500t + 82000 \Rightarrow = -11500 \times 15 + 82000 \Rightarrow y =$ – \$90500

(c) Comment on your answers in (b).

The first answer seems reasonable, but the second answer is unrealistic. The model cannot be valid when used for predicting many years into the future. The relationship between these variables for longer periods of time must become non-linear.

When information is collected in a real-life business situation, the data will not usually make a perfect straight line when we graph it. Models are not usually exact. But we can sometimes fit a line reasonably well to the data. The line may not pass through all the points, or even any of them. However, the points may all be reasonably close to the line. The data may show a trend, and the trend line becomes the model. This kind of analysis will be covered in Linear regression and Correlation Analysis section.